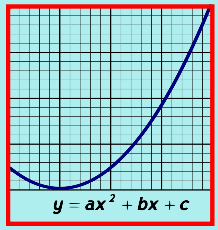
Math 25

Fall 2017

Lecture 1



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Math 25

College Algebra

Saturdays

8:00-11:50 AM

office hours

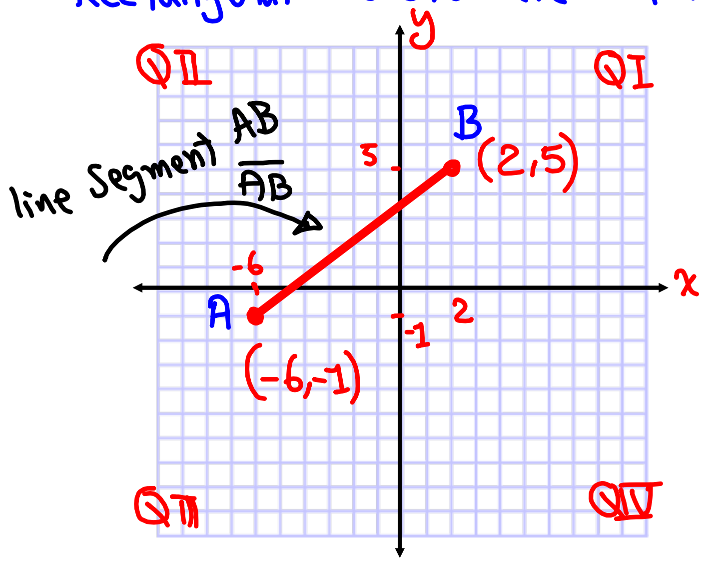
7:30-8:00

in our classroom.

I started Chaffey college

Fall 1992.

Rectangular Coordinate system



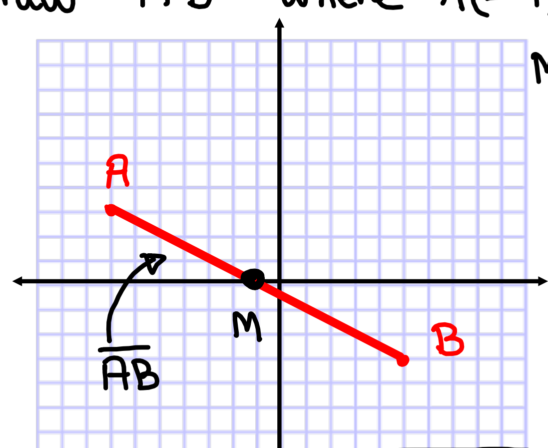
Ordered-Pair

(x, y)

Plot $(2,5)$

Plot $(-6,-1)$

Draw $\overline{AB}$ where $A(-7,3)$ & $B(5,-3)$



Midpoint $M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Slope $m = \dfrac{y_1 - y_2}{x_1 - x_2}$

distance

$$d = \sqrt{(x_1-x_2)^2 + (y_1-y_2)^2}$$

$$M\left(\frac{-7+5}{2}, \frac{3+-3}{2}\right) = \boxed{M(-1,0)}$$

Rise

Slope $m = \dfrac{3-(-3)}{-7-5} = \dfrac{6}{-12} = \boxed{\dfrac{-1}{2}}$

Run

distance $d = \sqrt{(-7-5)^2 + (3--3)^2} = \sqrt{(-12)^2+6^2} = \sqrt{144+36} = \sqrt{180} \approx \boxed{13.4}$

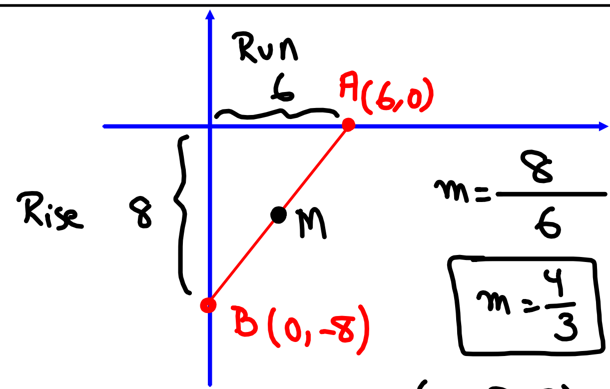
$A(6,0)$, $B(0,-8)$

1) Draw $\overline{AB}$

2) Find its midpoint

3) Find its slope

4) Find distance from A to B.



$m = \frac{8}{6}$

$\boxed{m = \frac{4}{3}}$

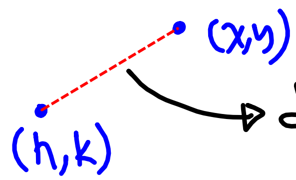
$M\left(\frac{0+6}{2}, \frac{-8+0}{2}\right)$

$\boxed{M(3,-4)}$

$$d = \sqrt{(0-6)^2 + (-8-0)^2} = \sqrt{(-6)^2 + (-8)^2}$$

$$= \sqrt{100} = \boxed{10}$$

Find the distance from (x,y) To (h,k)



$$d = \sqrt{(x-h)^2 + (y-k)^2}$$

If we assume that r is the distance,

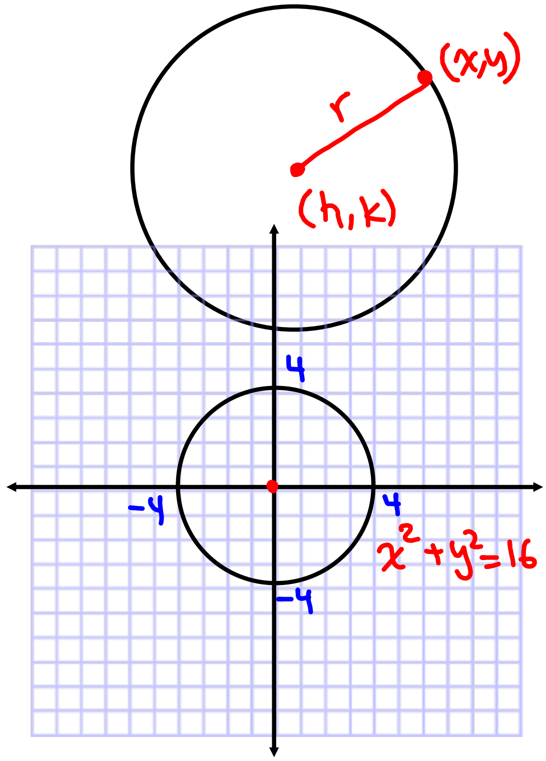
$$r = \sqrt{(x-h)^2 + (y-k)^2}$$

To remove the radical, we can Square both Sides.

$$r^2 = (x-h)^2 + (y-k)^2$$

$$(x-h)^2 + (y-k)^2 = r^2$$

Circle centered at (h,k) with radius r.



$(x-h)^2+(y-k)^2=r^2$

Consider

$x^2+y^2=16$

$(x-0)^2+(y-0)^2=4^2$

Center $(0,0)$, $r=4$

Domain → x-Values

Range → y-Values

D: $[-4,4]$

R: $[-4,4]$

Consider $(x+3)^2+(y-4)^2=25$

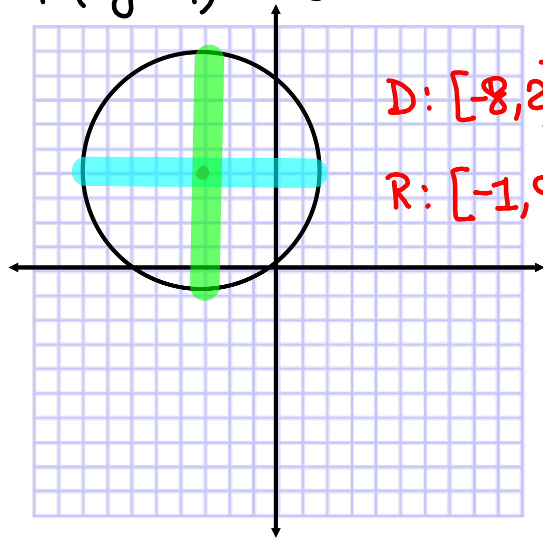
1) Center $(-3,4)$

2) Radius $r=5$

3) Draw it.

4) Domain

5) Range



D: $[-8,2]$

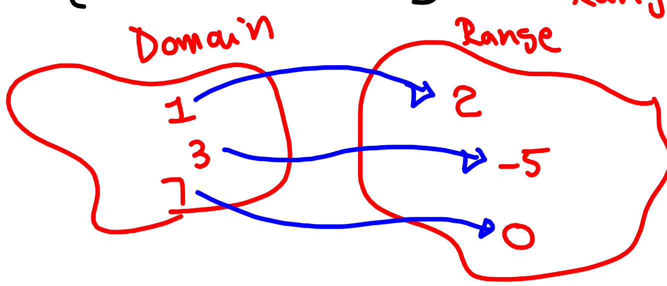
R: $[-1,9]$

Relations : It is a relationship that exists for a set of ordered-Pairs

$\{(1,2), (3,-5), (7,0)\}$

Domain $\{1,3,7\}$

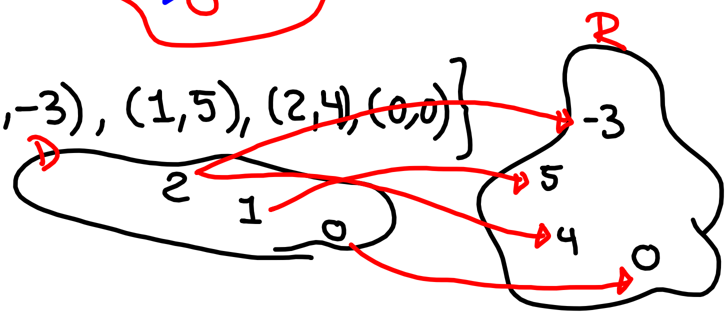
Range $\{2,-5,0\}$



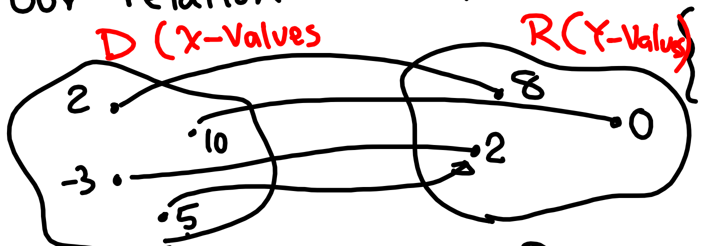
Consider $\{(2,-3), (1,5), (2,4), (0,0)\}$

D: $\{2,1,0\}$

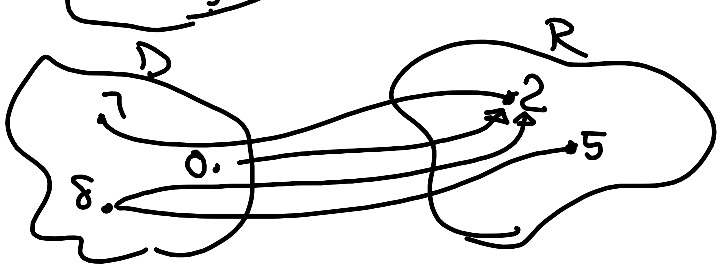
R: $\{-3,5,4,0\}$



Whenever any value from the domain matches with one element from the range, then our relation is called a function.



$\{(2,8), (10,0), (-3,2), (5,2)\}$ is a function.



$\{(7,2), (0,2), (8,2), (8,5)\}$

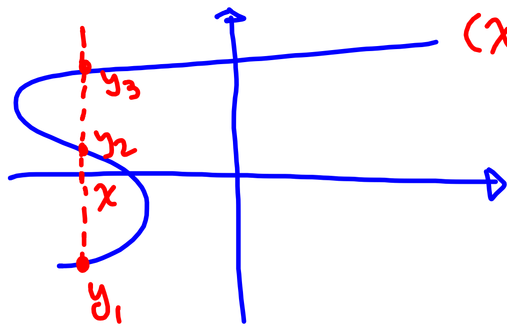
Not a function

$(8,5)$ & $(8,2)$

Every function is a relation.

Not every relation is a function.

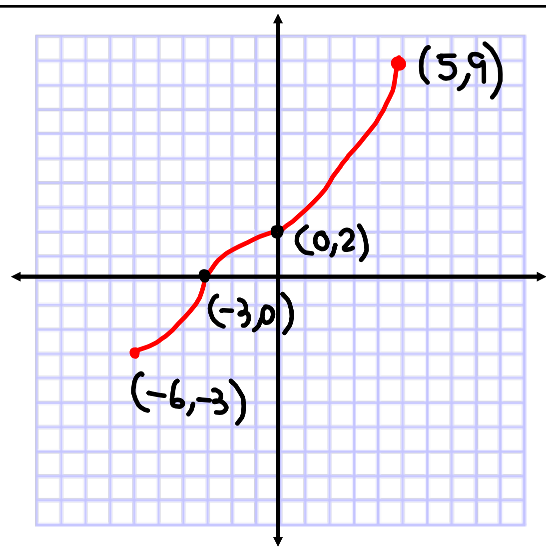
To determine whether a graph is a function or not, we use vertical line test



(x,y_1) (x,y_2) (x,y_3)

Same x, Different Y's.

Not a function.



① function or not?

Explain, Function, by V.L.T.

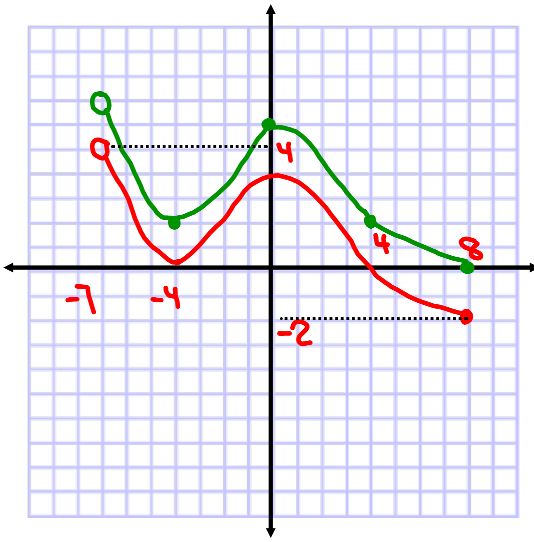
② Domain $[-6, 5]$

③ Range $[-3, 9]$

④ Any intercepts

x-Int $(-3,0)$

Y-Int $(0,2)$



① function or not?

Reason Function, V.L.T.

② Domain $(-7, 8]$

③ Range $[-2, 5)$

④ All intercepts

Y-Int $(0,4)$

x-Int $(-4,0), (4,0)$

⑤ shift this graph 2 units up, draw.

Recall $3x - 5y = 10$ Equation of a line

Isolate y

$-5y = -3x + 10$

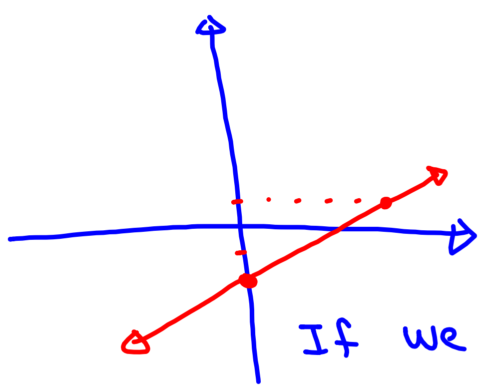
Graph it

$y = \frac{-3}{-5}x + \frac{10}{-5}$

$y = \frac{3}{5}x - 2$ Slope-Int form

$y = mx + b$

$m = \frac{3}{5}$ Y-Int $(0, -2)$



If we replace y with $f(x)$ "f of x"

we have function notation.

$\boxed{f(x) = \frac{3}{5}x - 2}$

$f(x) = x^2 - 4x$

Find $f(0)$, $f(2)$, $f(-2)$

$f(0) = 0^2 - 4(0) = 0 \Rightarrow (0, 0)$

$f(2) = 2^2 - 4(2) = 4 - 8 = -4 \Rightarrow (2, -4)$

$f(-2) = (-2)^2 - 4(-2) = 4 + 8 = 12 \Rightarrow (-2, 12)$

$f(x) = \dfrac{x+3}{x-2}$ Find $f(0)$, $f(2)$, and $f(-3)$

$f(0) = \dfrac{0+3}{0-2} = \boxed{\dfrac{-3}{2}}$

$f(2) = \dfrac{2+3}{2-2} = \dfrac{5}{0}$ $\boxed{\text{undefined}}$

$f(-3) = \dfrac{-3+3}{-3-2} = \dfrac{0}{-5} = \boxed{0}$

Some Special Functions

1) Constant Function $f(x) = b$

2) Linear Function $f(x) = mx + b$

3) Polynomial Function $f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots$

4) Rational Function $f(x) = \dfrac{\text{Polynomial}}{\text{Polynomial}}$

5) Square-Root Function $f(x) = \sqrt{x}$

6) Absolute Value Function $f(x) = |x|$

$f(x) = \sqrt{x} + 2$

we cannot take square-root of negative numbers, therefore $x \geq 0$ Domain $[0, \infty)$

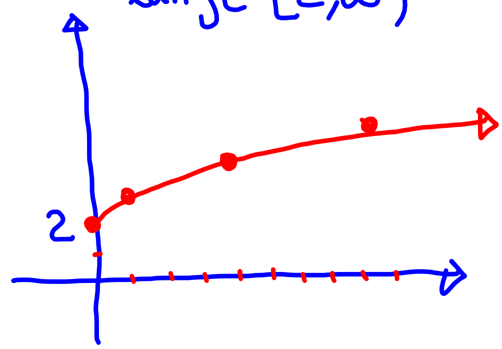
$f(0) = \sqrt{0} + 2 = 2$

$f(1) = \sqrt{1} + 2 = 3$

$f(4) = \sqrt{4} + 2 = 4$

$f(9) = \sqrt{9} + 2 = 5$

Range $[2, \infty)$



$f(x) = |x| - 4$ we can take abs. value of any number $\Rightarrow$ Domain $(-\infty, \infty)$

$f(0) = -4$

$f(2) = -2$

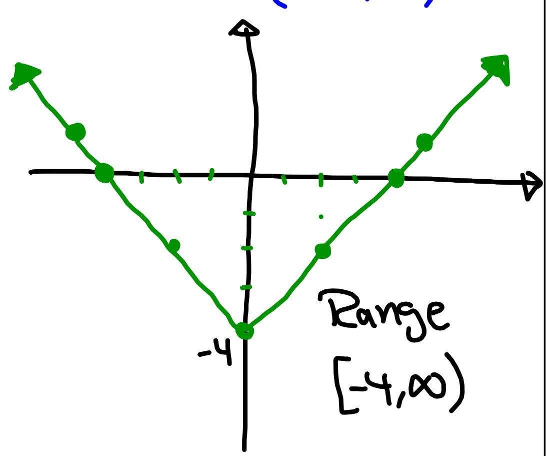
$f(4) = 0$

$f(-2) = -2$

$f(-4) = 0$

$f(5) = 1$

$f(-5) = 1$



Range $[-4, \infty)$

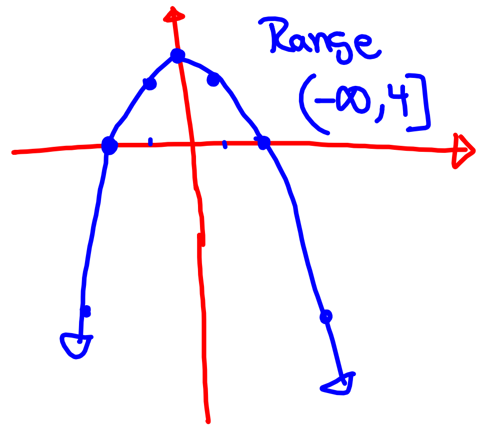
$f(x) = 4 - x^2$ Polynomial function,

$f(0) = 4$ $f(2) = 0$ we can use any value for x.

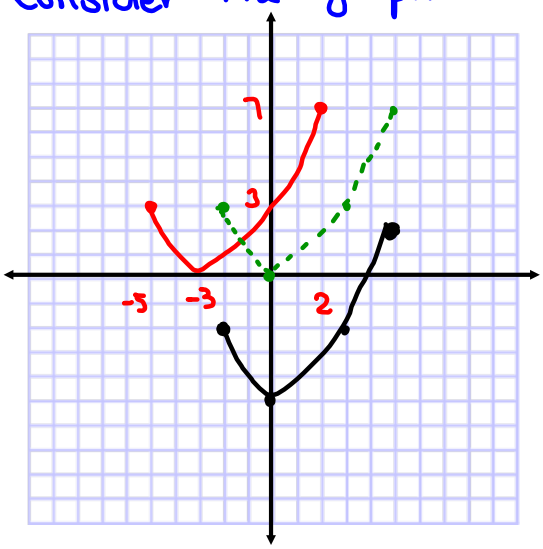
$f(1) = 3$ $f(-2) = 0$ Domain: $(-\infty, \infty)$

$f(-1) = 3$ $f(3) = -5$

$f(-3) = -5$



Consider the graph below



1) function or not?

function

2) Domain & Range

$[-5, 2]$ $[0, 7]$

3) All intercepts

x-Int $(-3, 0)$, Y-Int $(0, 3)$

4) shift the graph 3 units to the right, and then 5 units down.

$f(x) = x^2$

find

$f(4)$, $f(x+h)$, and $f(3x)$

$f(4) = 4^2 = \boxed{16}$

$f(x+h) = (x+h)^2$

$= (x+h)(x+h)$

$= \boxed{x^2 + 2xh + h^2}$

$f(3x) = (3x)^2$

$= \boxed{9x^2}$

$f(x) = 3x - 2$

find

1) $f(x+h) = 3(x+h) - 2 = 3x + 3h - 2$

2) Simplify $\dfrac{f(x+h) - f(x)}{h}$

$= \dfrac{3x + 3h - 2 - (3x - 2)}{h}$

$= \dfrac{\cancel{3x} + 3h \cancel{- 2} \cancel{- 3x} \cancel{+ 2}}{h} = \dfrac{3\cancel{h}}{\cancel{h}} = \boxed{3}$

Find the domain

$f(x) = 2x - 5$

Linear Function

$(-\infty, \infty)$

$g(x) = -8$

Constant Function

$(-\infty, \infty)$

$h(x) = \dfrac{x-2}{x+5}$

Rational Function

$x + 5 \neq 0$

$x \neq -5$

All reals except -5

$k(x) = \dfrac{x}{x^2 - 9}$

Rational Function

$x^2 - 9 \neq 0$

$x^2 \neq 9$

$x \neq \pm 3$

All Reals except ± 3

$f(x) = \dfrac{x+4}{x^2 - 4x - 12}$

Find $f(0)$, $f(-4)$, and then discuss domain.

$f(0) = \dfrac{0+4}{0^2 - 4(0) - 12} = \dfrac{4}{-12}$

$\boxed{f(0) = \frac{-1}{3}}$

$f(-4) = \dfrac{-4+4}{(-4)^2 - 4(-4) - 12} = \dfrac{0}{20} = \boxed{0}$

Rational Function

$x^2 - 4x - 12 \neq 0$

$(x-6)(x+2) \neq 0$

$x \neq 6$, $x \neq -2$

Domain: All Reals except 6 & -2.

$$f(x)=\frac{\sqrt{x}-2}{x^2+2x+2}$$

Find $f(0)$, $f(4)$, Discuss domain.

$f(0)=-1$

$f(4)=0$

$x^2+2x+2\neq 0$, Also $x\geq 0$

$\underbrace{x^2+2x+1}+1\neq 0$

$(x+1)^2+1\neq 0$

No real Solution

$[0,\infty)$

Graph below belongs to

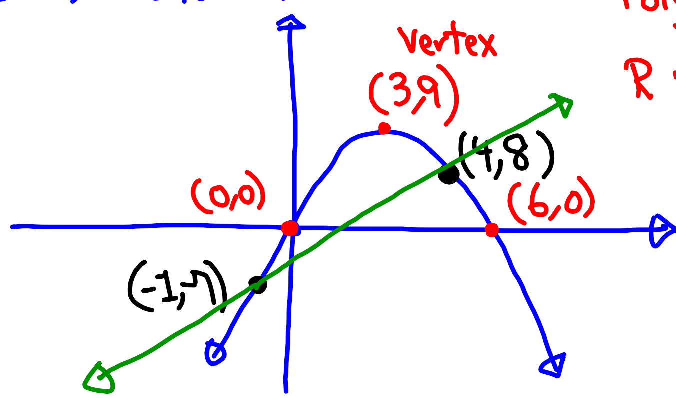
$f(x)=6x-x^2$

D: $(-\infty,\infty)$

Polynomial function

R: $(-\infty,9]$

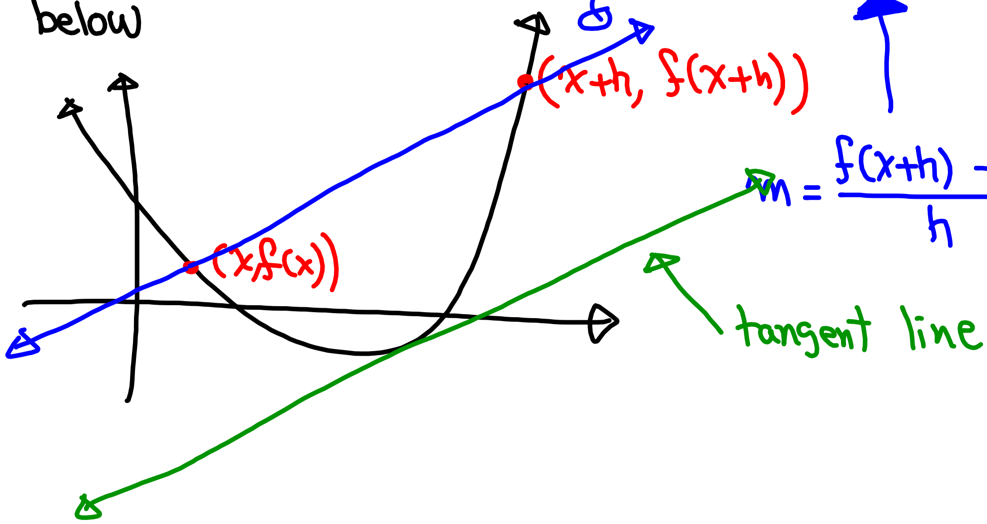
Find $f(-1)$ & $f(4)$



Secant line, find its slope

$$m=\frac{8-(-7)}{4-(-1)}=\frac{15}{5}=\boxed{3}$$

Consider the graph below



$$m = \frac{f(x+h) - f(x)}{h}$$

$f(x) = x^2 + 5$

find $f(2)$, $f(x+2)$

$f(2) = 2^2 + 5 = \boxed{9}$

$f(x+2) = (x+2)^2 + 5$

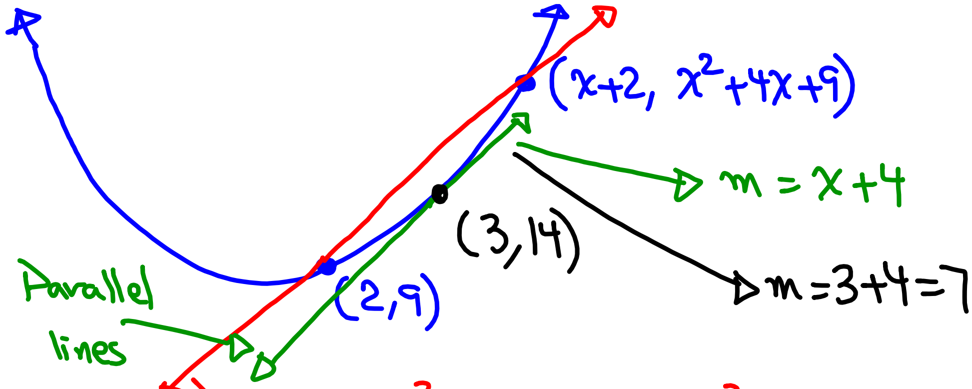
$= x^2 + 4x + 4 + 5$

$= x^2 + 4x + 9$

Compute $\dfrac{f(x+2) - f(2)}{x}$

$$= \frac{x^2 + 4x + 9 - 9}{x} = \frac{x^2 + 4x}{x} = \frac{\cancel{x}(x+4)}{\cancel{x}}$$

$$= \boxed{x+4}$$



$$m = \frac{x^2+4x+9-9}{x+2-2} = \frac{x^2+4x}{x} = x+4$$

$f(x) = \sqrt{x}$

Find $f(h+4)$, $f(4)$

$= \sqrt{h+4}$ $\quad = 2$

$5 = \sqrt{25} = \sqrt{16+9}$

$\neq \sqrt{16} + \sqrt{9} = 4+3$

$= 7$

Compute & Simplify

$$\frac{f(h+4) - f(4)}{h} = \frac{\sqrt{h+4} - 2}{h}$$

$$= \frac{\sqrt{h+4} - 2}{h} \cdot \frac{\sqrt{h+4} + 2}{\sqrt{h+4} + 2} = \frac{(\sqrt{h+4})^2 - (2)^2}{h(\sqrt{h+4} + 2)}$$

$(A-B)(A+B) = A^2 - B^2$

$$= \frac{\cancel{h}\overset{1}{} \cancel{+4} \cancel{-4}}{\cancel{h}(\sqrt{h+4} + 2)} = \boxed{\frac{1}{\sqrt{h+4} + 2}}$$

$$f(x)=\frac{x^2-4}{x^2-5x+6}$$

Find $f(2)=\frac{2^2-4}{2^2-5(2)+6}=\frac{0}{0}$

Indeterminate form

This concept along with some condition is used in calc. for limits.

Simplify $f(x)$

$$=\frac{(x+2)\cancel{(x-2)}}{(x-3)\cancel{(x-2)}}=\frac{x+2}{x-3}$$

Now find $f(2)$

$$f(2)=\frac{2+2}{2-3}=\frac{4}{-1}=-4$$

Looking Ahead:

$n!$

"n factorial"

$n!=n(n-1)(n-2)\cdots\cdots 3\cdot 2\cdot 1$

$5!=5\cdot 4\cdot 3\cdot 2\cdot 1=\boxed{120}$

$8!=8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1=$

$\boxed{40320}$

By Definition $0!=1$

Simplify $\frac{7!}{5!}=\frac{7\cdot 6\cdot \cancel{5\cdot 4\cdot 3\cdot 2\cdot 1}}{\underset{1}{\cancel{5\cdot 4\cdot 3\cdot 2\cdot 1}}}=\boxed{42}$

Simplify $\dfrac{8!}{5!\cdot 3!} = \dfrac{8\cdot 7\cdot \cancel{6}\cdot \cancel{5!}}{\cancel{5!}\cdot \cancel{3\cdot 2\cdot 1}} = \boxed{56}$

Combination Formula $\quad {}_nC_r = \dfrac{n!}{r!\cdot(n-r)!}$

Find ${}_7C_3$

$${}_7C_3 = \frac{7!}{3!\cdot(7-3)!}$$

$$= \frac{7!}{3!\cdot 4!}$$

$$= \frac{7\cdot \cancel{6}\cdot 5\cdot \cancel{4!}}{\cancel{3\cdot 2}\cdot 1\cdot \cancel{4!}}$$

$$= \boxed{35}$$

$${}_{50}C_5 = \frac{50!}{5!\cdot(50-5)!} = \frac{50!}{5!\cdot 45!}$$

$$= \frac{\overset{10}{\cancel{50}}\cdot 49\cdot \overset{2}{\cancel{48}}\cdot 47\cdot 46\cdot \cancel{45!}}{\cancel{5}\cdot \cancel{4}\cdot \cancel{3}\cdot \cancel{2}\cdot 1\cdot \cancel{45!}}$$

$$= 10\cdot 49\cdot 2\cdot 47\cdot 46$$

$$= \boxed{2,118,760}$$

Permutation formula

$$_nP_r = \frac{n!}{(n-r)!}$$

$$_8P_3 = \frac{8!}{(8-3)!}$$

$$= \frac{8!}{5!}$$

$$= \frac{8 \cdot 7 \cdot 6 \cdot 5!}{5!}$$

$$= \boxed{336}$$